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## Prediction of the fracture in cold forging with modified Cockcroft-Latham criterion

Sergey Stebunov<sup>a\*</sup>, Andrey Vlasov<sup>b</sup>, Nikolai Biba<sup>c</sup>

<sup>a</sup>QuantorForm Ltd. P.O. Box 74, Moscow, 115088, Russia

<sup>b</sup>Bauman Moscow State Technical University, ul. Baumanskaya 2-ya, 5, Moscow, 105005, Russia

<sup>c</sup>Micas Simulation Ltd., 107 Oxford Road, Oxford, OX4 2ER, UK

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### Abstract

Cold forging technology provides production of net shape parts that required very little or no further machining. Meanwhile one of the technological problems of the cold forging is fracture of the workpiece during production. This is especially important when using less ductile high strength materials or technological schemes that require significant deformation. The use of reliable fracture criteria implemented in metal forming software can help predict possible cracks and find ways to avoid them. There are many criteria that can be used for this task, for example, Gurson and its modifications, Johnson-Cook, Wierzbicki and Cockcroft-Latham. The latter one is the most widely used due to its simplicity since only the first principal component of stress is required for calculation. On the other hand, in this simple form, it does not take into consideration the influence of the stress state and the path of plastic deformation. In the presented work, a comparative analysis of different criteria has been done to find out the relations among them and a modified Cockcroft-Latham criterion of fracture has been proposed. It considers both the influence of the normalized first principal stress and the influence of the stress state scheme. A new approximation of the surface of critical plasticity has also been proposed that considers the limit values of the normalized average stress. The modified Cockcroft-Latham criterion was programmed as a subroutine in QForm metal forming simulation software and it has shown good accuracy in tests for different practical cold forming processes.

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\* Corresponding author. Tel.: +7-916-684-7463; fax: +7-495-663-3380.

E-mail address: [serg@qform3d.com](mailto:serg@qform3d.com)

## 1. Introduction

The following fracture criteria are widely used to predict the material failure during plastic forming: Gurson [1, 2], Johnson-Cook [3], Bai-Wierzbicki [4] and Cockcroft-Latham [5]. The latter one is the most popular among them. The analysis of publications on the Internet over the last 5 years has shown that the Cockcroft-Latham criterion is mentioned more than 2000 times in the scientific and technical literature. The main reason of this success is its simplicity and very little material data required for calculations. The serious drawback of this criterion that it does not take in consideration the influence of the stress state and the strain path during deformation.

Some modifications of the Cockcroft-Latham criterion try to overcome these drawbacks as, for example, the variant proposed in work [6] that is based on their hypothesis of the linear accumulation of damage. That proposed modification allows predicting of the failure of the deformed material in case when the stress-strain state changes during forging process. However, that model does not take into account the critical plastic strain dependence on the type of the stress state that in turn has been experimentally observed and proved by several authors [7,4].

The idea of the presented work is to find out the limits of applicability of standard Cockcroft-Latham criterion and its similarity to other criteria with the purpose to find an appropriate modification of the criterion that takes into account the stress state scheme and its influence on the critical plastic strain.

## 2. The normalized criterion of Cockcroft-Latham-Oh

The Cockcroft-Latham criterion is based on the hypothesis that the accumulation of damages occurs only when at least one of the principal stress component is tensile. Subsequently in work [8] the criterion was modified by normalizing the first principal stress  $\sigma_1^+$  with respect to effective stress  $\bar{\sigma}$ .

$$\int_0^{\bar{\varepsilon}_p} \frac{\sigma_1^+}{\bar{\sigma}} d\bar{\varepsilon}_p < C_1, \quad (1)$$

where  $d\bar{\varepsilon}_p$  is the increment of the plastic strain,  $C_1$  is the parameter that is usually called material constant,  $\bar{\varepsilon}_p$  is the effective plastic strain. The expression (1) was called the "normalized criterion of Cockcroft-Latham-Oh" and many researchers use it for prediction of damage in cold metal forming. Let us show that the critical value  $C_1$  for the normalized Cockcroft-Latham-Oh criterion is not a material constant but it depends at least on the stress state.

The first principal stress equals the effective stress in the case of uniaxial tension  $\sigma_1^+ = \bar{\sigma}$ . Then the critical strain due to equation (1) exactly equals the plastic strain that is accumulated till the damage.

$$C_{1u} = \int_0^{\bar{\varepsilon}_f} \frac{\sigma_1^+}{\bar{\sigma}} d\bar{\varepsilon}_p = \int_0^{\bar{\varepsilon}_f} d\bar{\varepsilon}_p = \bar{\varepsilon}_{f \text{ ultimate}}. \quad (2)$$

In the case of the pure shearing or plane-strain state is valid the expression  $\sqrt{3}\sigma_1^+ = \bar{\sigma}$ . The critical stress that is accumulated till the damage for the pure shearing equals:

$$C_{1s} = \int_0^{\bar{\varepsilon}_f} \frac{\sigma_1^+}{\bar{\sigma}} d\bar{\varepsilon}_p = \int_0^{\bar{\varepsilon}_f} \sqrt{3} d\bar{\varepsilon}_p = \sqrt{3} \bar{\varepsilon}_{f \text{ shear}}. \quad (3)$$

If we assume that the critical value  $C_1$  is the material constant and compare Eqs. (2) and (3) we may see that plastic strain at the moment of the damage in tension conditions always bigger  $\sqrt{3}$  times than in shearing. This is not confirmed by the experimental data. The stress state in any technological processes in every material particle changes continuously. It means that the critical value of the damage in the formula (1) should be dependent on the strain path.

Thus, the normalized criterion (1) does not allow a quantitative prediction of the workpiece damage for an arbitrary technological process of cold forging. The criterion can be used only for a qualitative estimation of similar technological processes in which the stress state and the strain paths for the critical points differ insignificantly.

### 3. Modified criterium Cockcroft-Latham-Oh

The modification of Cockcroft-Latham-Oh criterion was proposed in work [6]. The proposed criterion is based on the following assumptions:

- The increment of the damage for a material point is proportional to the increment of the plastic strain as in (1) and inversely proportional to the critical value of the strain (damage strain) that in turn depends on the stress state.
- In addition, the increment of damage in the material point does not depend on the strain path, which is equivalent to the hypothesis of linear accumulation of damage increments.

Assuming that the increments of the plastic deformation are sufficiently small to neglect the changing of the normalized stress, the modified criterion can be converted to the expression:

$$\psi = \int_0^{\bar{\varepsilon}_p} \frac{d\bar{\varepsilon}_p}{\bar{\varepsilon}_f(\chi)}, \quad (4)$$

where  $\psi$  is the damage factor,  $\chi = \sigma_1^+/\bar{\sigma}$  is the normalized first principal stress,  $\bar{\varepsilon}_f(\chi)$  is the critical strain that corresponds to the material damage in tests with constant value of  $\chi$ .

Similar methods of estimation the damage of the material in plastic deformation have been proposed in the works [3,4,7] and some others.

These criteria are characterized by the following basic features:

1. The critical strain is dependent not on the normalized first principal stress  $\chi = \sigma_1^+/\bar{\sigma}$ , but on the normalized mean stress (stress triaxiality)  $\eta = \sigma_m/\bar{\sigma}$ .
2. The critical strain is dependent on stress state which is characterised by various invariant parameters such as Lode parameter  $\mu_\sigma$ , normalized Lode angle  $\bar{\theta}$  or normalized third invariant of stress deviator  $\xi$ .

The dependence of the critical strain on triaxiality and the stress state which is function of two arguments that can be represented as a surface. In the work [4], the normalized Lode angle is used as a parameter characterizing the stress state. Its value is determined by the equation:

$$\bar{\theta} = 1 - \frac{2}{\pi} \arccos \xi, \quad (5)$$

where  $\xi = \frac{27J_3}{2\bar{\sigma}^3}$  is the normalized third invariant of stress deviator  $J_3 = (\sigma_1 - \sigma_m)(\sigma_2 - \sigma_m)(\sigma_3 - \sigma_m)$ ,  $\sigma_1, \sigma_2, \sigma_3$  is the principal stresses,  $\sigma_m$  is the mean stress. Lode parameter  $\mu_\sigma$  is related to the normalized third invariant of the stress deviator  $\xi$  by the following relationship [9]

$$\mu_\sigma = \sqrt{3} \tan \left( -\frac{1}{3} \arcsin \xi \right). \quad (6)$$

The function of the critical strain in work [4] is approximated by parabolic function as follows:

$$\bar{\varepsilon}_f = a_0(\eta) + a_1(\eta) \cdot \bar{\theta} + a_2(\eta) \cdot \bar{\theta}^2, \quad (7)$$

where  $a_0, a_1, a_2$  are the empirical coefficients dependent on triaxiality.

It should be mentioned that similar expression was previously independently proposed in work [10]:

$$\Lambda_f = b_0(K) + b_1(K) \cdot \mu_\sigma + b_2(K) \cdot \mu_\sigma^2, \quad (8)$$

where  $b_0, b_1, b_2$  are the empirical coefficients,  $\Lambda_f = \sqrt{3}\bar{\varepsilon}_f$  is the accumulated critical shearing strain,  $K = \sigma_m/T = \sqrt{3}\eta$  is the stress state coefficient,  $T$  is the shearing yield stress.

The normalized Lode angle  $\bar{\theta}$  equals Lode parameter  $\mu_\sigma$  with the opposite sign [10]. Thus, there is no the principal difference between Eqs. (7) and (8).

The experimental curves of the critical strain are approximated by the exponential expression as follows:

$$\bar{\varepsilon}_f = M_1 \exp(M_2 \cdot \eta). \quad (9)$$

The coefficients  $M_i$  in Eq. (9) are defined by approximation of the experimental tests.

The parameters  $\eta, \chi, \mu_\sigma$  are related to each other by the following relationship as shown in work [12]:

$$\frac{\sigma_m}{\sigma} = \frac{\sigma_1}{\sigma} - \frac{3 - \mu_\sigma}{3\sqrt{3 + \mu_\sigma^2}} \quad \chi = \eta + \frac{3 - \mu_\sigma}{3\sqrt{3 + \mu_\sigma^2}} \quad \text{or} \quad (10)$$

As we can see there is one-to-one correspondence between the critical plasticity function in the coordinates  $\eta, \bar{\theta}$  and  $\chi, \mu_\sigma$ . Thus, the function in formula (4) can be replaced by the expression that takes into account the type of the stressed state, for example,  $\bar{\varepsilon}_f(\chi, \mu_\sigma)$ . Then finally the expression for the evolution of the damage in the modified Cockcroft-Latham-Oh criterion can be presented as follows:

$$\psi = \int_0^{\bar{\varepsilon}_p} \frac{d\bar{\varepsilon}_p}{\bar{\varepsilon}_f^c(\chi, \mu_\sigma)}, \quad (11)$$

where  $\bar{\varepsilon}_f^c(\chi, \mu_\sigma)$  is the critical strain dependent on  $\chi, \mu_\sigma$ . According to Eq. (10)  $\chi = \varphi(\eta, \mu_\sigma)$ , we can conclude that the modified Cockcroft-Latham-Oh criterion indirectly takes into account the influence of the middle principal stress on the damage calculation.

Let us analyse the relationship between the proposed equation of damage of Eq. (11) and equations in Bogatov's criteria [7] and Bai-Wierzbicki [4], which in our notation looks as:

$$D = \int_0^{\bar{\varepsilon}_p} \frac{d\bar{\varepsilon}_p}{\bar{\varepsilon}_f^b(\eta, \bar{\theta})}, \quad (12)$$

where  $\bar{\varepsilon}_f^b(\eta, \bar{\theta})$  is the critical strain dependent on  $\eta, \bar{\theta}$ . Obviously, if the critical plasticity function in coordinates  $\eta, \bar{\theta}$  is represented in coordinates  $\chi, \mu_\sigma$ , then the accumulated damage by Eqs. (11) and (12) should provide the same results in the zone of positive values of the first principal stress. In the case of negative first principal stresses, according to the Cockcroft-Latham hypothesis, the increase in damage will be stopped, therefore calculation by criterion (11) gives a smaller result than Eq. (12).

#### 4. Approximation of the critical strain surface and limit value of stress triaxiality for ductile fracture

The idea that the ductile fracture does not occur if the stress triaxiality is less than  $\eta_{crit} = -0.33$  was proposed in work [13]. This idea is based on Bridgeman experiments. Similar effect is described in the work [7] although with the remark that this phenomenon is not valid for all materials.

It is shown in work [11] that the critical stress triaxiality for the different types of stress state is different. For example, in uniaxial tension with applied hydrostatic pressure  $\eta_{crit} = -2/3$ , in plane-strain deformation  $\eta_{crit} = -1/\sqrt{3}$ , in uniaxial compression test  $\eta_{crit} = -1/3$ .

Accepting the validity of the Cockcroft-Latham hypothesis, it should be concluded that the critical plastic strain should asymptotically approaches the vertical axis when the stress triaxiality approaches the limiting value. However, the exponential approximation of the critical strain does not satisfy this condition. The condition of asymptotical approach can be satisfied using the approximation based on logarithmic expression:

$$\bar{\varepsilon}_f^c(\eta, \mu_\sigma) = q_0(\mu_\sigma) + q_1(\mu_\sigma) \cdot \ln[\eta + \eta_{crit}(\mu_\sigma)], \quad (13)$$

where  $q_0$   $q_1$   $q_2$  are the empirical coefficients dependent on Lode index and  $\chi = \varphi(\eta, \mu_\sigma)$  according to Eq. (10).

In the Fig.1 is shown the comparison of the exponential approximation according to Eq. (9) and logarithmic approximation according to formula (13) for the experimental results of torsion test ( $\mu_\sigma = 0$ ) in the high-pressure chamber of X12CrNiMo 19-10 (DIN) steel presented in work [7].

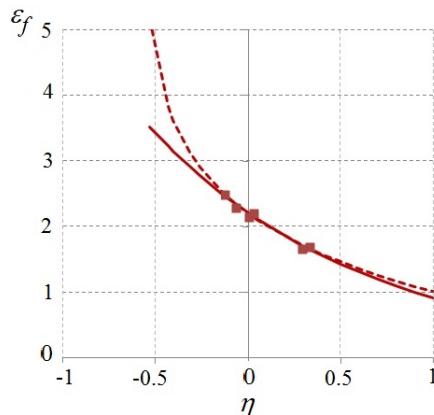


Fig. 1. Approximation of the torsion test [7] for steel X12CrNiMo 19-10 (DIN). Solid line is the exponential approximation according to formula (9) and dashed line is the logarithmic approximation according to formula (13). Dots are experimental data.

We can see from the Fig.1 that in the points of the tests the both approximations are practically identical. The increasing of the stress triaxiality in the positive direction does not cause big difference in the critical strain. Meanwhile at the same time near the compressive limit of the stress triaxiality  $\eta = -0.5$  the difference in the critical strain  $\bar{\varepsilon}_f$  considerably increases.

The functions in Eq. (13) may be approximated by parabolic function using the critical strain curves that are obtained from the tests like uniaxial compression, tension and torsion. The transformed surface of the critical strain for C15 (DIN) tested in work [7] using this approach compared with initial exponential approximation is shown in the Fig.2.

The numbers on the curves shows the critical plastic strain values that can be distinguished in the tests with constant values the parameters  $\eta, \mu_\sigma$ . It is obvious that for tensile stress state both surfaces show the same or very close values. Meanwhile if the values of  $\eta_{crit}(\mu_\sigma)$  become smaller than -0.33 the plasticity of the material rapidly increases as shown in Fig. 2b that is confirmed by the results of works [10,12].

The calculation of the critical plasticity using Bai-Wierzbicki criterion (12) with approximation the critical strain as in Eq. (13) causes increasing of the plasticity in the area of the compressed stress state and leaves the prediction of the plasticity unchanged in the area of tensile stress state.

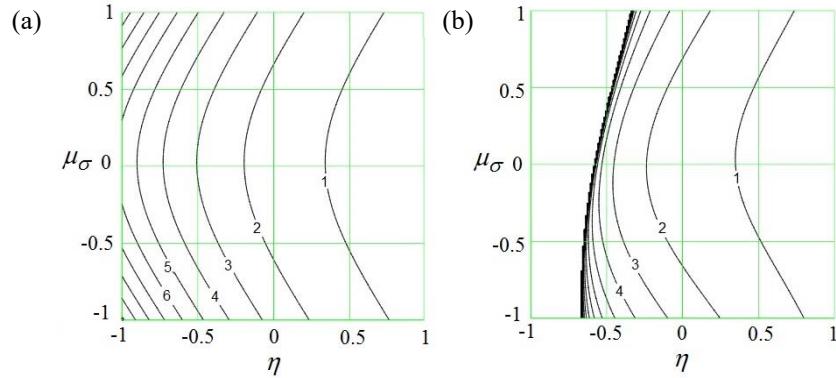


Fig. 2. Isolines of the critical strain for steel C15 (DIN). (a) Exponential approximation from [7], (b) logarithmic approximation according to Eq. (13).

## 5. Examples of damage prediction by modified criteria Cockcroft- Latham-Oh

The modified criterion Cockcroft-Latham-Oh was used for development of the user's subroutine and predicting the workpiece damage in several technological processes with the help QForm metal forming simulation software [14]. The first considered technology is described in work [15]. The sequence of the technological operations for cold forging of the cap made of steel C40 (DIN) includes 3 operations. The workpiece damage happens in the 3<sup>rd</sup> operation and is shown in the Fig. 3(a).

The technology was analysed by implementing the modified criterion Cockcroft- Latham-Oh following formula (11) and normalized criterion Cockcroft- Latham-Oh as in Eq. (1). The workpiece material is simulated as rigid-viscoplastic material. The flow stress of the steel C40 was approximated by Hensel-Spittel formula with coefficients shown in work [16]. The friction on the contact between the workpiece and tools was approximated by Levanov friction law [17] with friction factor 0.2. The heat transfer coefficient between workpiece and tool during simulation was specified as 40000W/m<sup>2</sup>K.

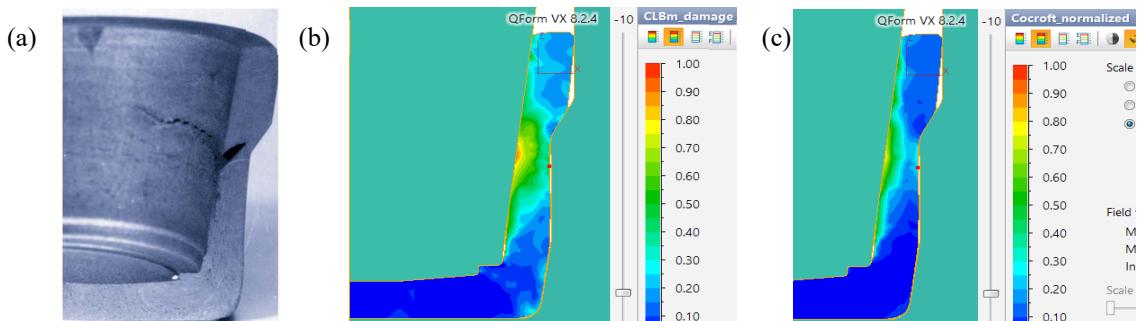


Fig. 3. Photo of failure of cold forged part made from steel 1040, DIN (a), damage criterion distribution in the workpiece in the third operation calculated using modified criterion Cockcroft- Latham-Oh (b), distribution of normalized criterion Cockcroft- Latham-Oh (c).

The distribution of the damage factor according to criterion (11) is shown in the Fig. 3(b). Its maximum value 0.93 indicates possible damage in the same area as in the Fig. 3(a). In the Fig. 3(c) is shown the distribution the normalized criterion Cockcroft- Latham-Oh (1). Its maximum value 0.84 also is located in the zone of actual fracture.

This example shows that in case monotonic deformation and absence of highly compressive stress state both criteria give similar results. Meanwhile in case of more complicated stress state the proposed modified criteria Cockcroft- Latham-Oh provides better results as shown below. Let us consider the cold forging technology of the part made of steel C40 (DIN). The sequence of the cold forging operation is shown in the Fig 5(a). The dimensions of the initial cylinder: height 22 mm and diameter 35 mm. At the end of the second operation the fracture appears on the side surface of the product as shown in the Fig.4(b). The material model and boundary conditions for simulation were similar to the first example that was shown in the Fig.3.

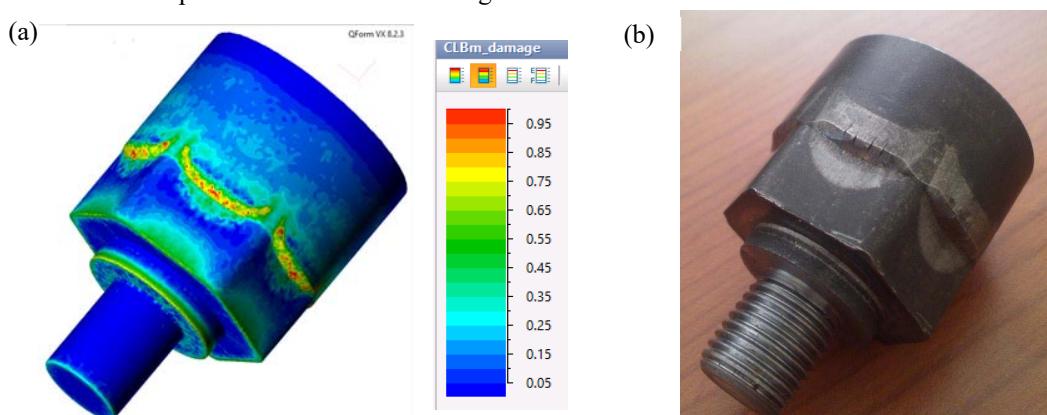


Fig. 4. (a) Damage factor distribution predicted by modified criterion Cockcroft- Latham-Oh and (b) actual cracks in the cold forged part.

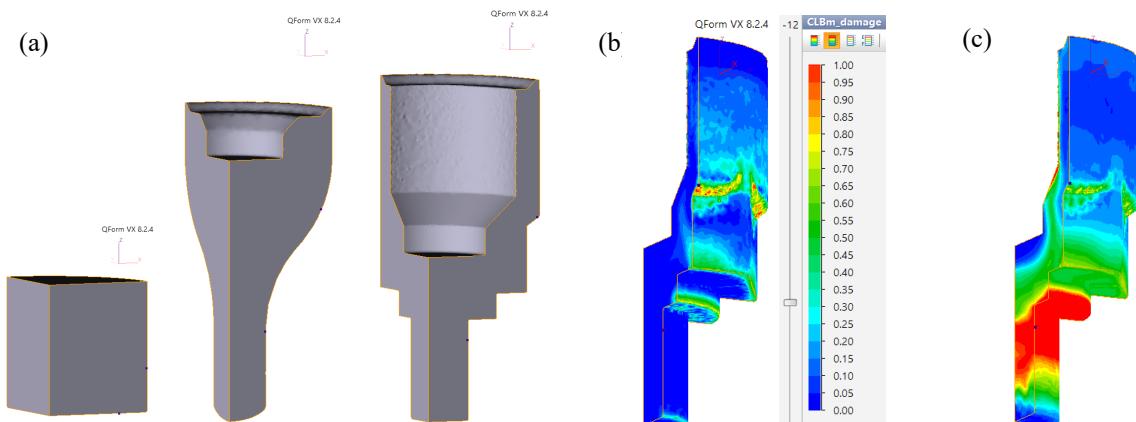


Fig 5. The sequence of cold forging operations (a) and comparison of two criteria distributions: (b) modified criterion Cockcroft- Latham-Oh (12); (c) normalized criterion Cockcroft- Latham-Oh (1).

Fig. 4(a) shows the predicted distribution of damage according to criterion (11). Fig.5 depicts the comparison of these two criteria distributions in the cross-section. Fig. 5(b) shows the modified criterion Cockcroft-Latham-Oh and in Fig. 5(c) is the normalized criterion Cockcroft-Latham-Oh. The considerable difference of the values and distributions of these two criteria can be explained by the complex path of the plastic strain and stress state

development during the forging operation. The simulation of the forging operation has shown very complicated path of the mean stress development in the critical area while at the bottom area of the part we observe only the minor increasing of the compressed stress state. As we see from this example the normalized Cockcroft- Latham-Oh criterion doesn't provide the realistic prediction of the ductile fracture in case of complex paths of the stress and strain that may develop in metal forming operations.

## 6. Conclusion

1. The normalized Cockcroft-Latham-Oh criterion does not provide quantitative prediction of material failure in some cold forging process. The criterion can be used only for a qualitative comparison of similar technological processes where the stress state and the strain path in dangerous points differ insignificantly.
2. It is shown that the critical value of the Cockcroft-Latham-Oh criterion is not just the characteristics of the material but it depends on the normalized first principal stress, the type of stress state and the strain path.
3. The proposed new modified Cockcroft-Latham-Oh criterion takes into account the effect of the normalized first principal stress and the type of stress state.
4. A new proposed approximation of critical strain takes into account the critical values of triaxiality.
5. The implementation of the proposed modified Cockcroft- Latham-Oh criterion to the workpiece fracture analysis in different technological processes has shown its quite accurate prediction of the damage.

## References

- [1] A. L. Gurson, Continuum theory of ductile rupture by void nucleation and growth: Part I. yield criteria and flow rules for porous ductile media, *Journal of Engineering Materials and Technology*, 99 (1977) 2–15.
- [2] A.V.Vlasov, D.A.Gerasimov, Realization of the Gurson-Tvergaard-Needleman model for calculation the processes of cold bulk forging of incompressible materials, *Izvestiya vysshikh uchebnykh zavedeniy. Mashinostroyeniye*, 8 (2017) 8-17. (In Russian).
- [3] G.R. Johnson, W.H. Cook, Fracture characteristics of three metals subjected to various strain, strain rates, temperatures, pressures, *Eng. Frac. Mech*, 21 (1985) 31 – 48.
- [4] Y. Bai, T. Wierzbicki, A new model of metal plasticity and fracture with pressure and Lode dependence, *International Journal of Plasticity* 24 (2008) 1071–1096.
- [5] M. G. Cockcroft, D. J. Latham, Ductility and the workability of metals, *J Inst Metals*, 96 (1968) 33-39.
- [6] A.V. Botkin, R.Z. Valiyev, Evaluation of metal damage during cold plastic deformation using the Cockcroft-Latham fracture model, *Deformatsiya i razrusheniye materialov*, 7 (2011) 17–22. (In Russian).
- [7] A. Bogatov, O. Mizhiritskiy, S.V. Smirnov, Resource of plasticity of metals in metal forming, (1984) 144. (In Russian).
- [8] S. I. Oh, C. C. Chen, S. Kobayashi, Ductile fracture in axisymmetric extrusion and drawing—part 2: workability in extrusion and drawing, *Journal of Engineering for Industry*, 101 (1979) 36-44.
- [9] L. Xue, T. Wierzbicki, Numerical simulation of fracture mode transition in ductile plates, *International Journal of Solids and Structures*, 46 (2009) 1423–1435.
- [10] S.V. Smirnov, N.V. Biba, S.A. Stebunov, Damage modelling in cold bulk metal forming using adaptive theory, *Proceedings of the 5th ESAFORM Conference on Material Forming*, (2002) 139-142.
- [11] Y. Bai, T. Wierzbicki, Comparative study of three groups of ductile fracture loci in the 3D space, *Engineering Fracture Mechanics*, 135 (2015) 147-167.
- [12] R. Sivak, I. Sivak, Plasticity of metals under complex loading, *Visnik Natsional'nogo tekhnichnogo universitetu Ukrainsi «Kiiv's'kij politekhnichniy institut»*. Seriya Mashinobuduvaniya,60 (2012) 129–132. (In Russian).
- [13] Y. Bao, T. Wierzbicki, On the cut-off value of negative triaxiality for fracture, *Engineering Fracture Mechanics*, 72 (2005) 1049-1069.
- [14] S. Stebunov, N. Biba, A. Vlasov, A. Maximov, Thermally and mechanical coupled simulation of metal forming processes, *Proceeding 10th Int. Conf. Techn. Plast. (ICTP)*, (2011) 171-175.
- [15] N. Biba, S. Stebunov, S. Smirnov, Application of adaptive damage theory for optimisation of cold bulk metal forming, *Proceedings of the 7th Int. Conf. on Num. Meth. in Ind. forming processes (NUMIFORM)*, (2001) 361-365.
- [16] A. Hensel, T. Spittel, Kraft- und Arbeitsbedarf bildsamer Formgebungsverfahren, (1978) 356. (in German).
- [17] A. Levanov, V. Kolmogorov, V. Burkin, Contact friction in metal forming processes, (1974) 416. (in Russian)